Differentiation is a method for calculating the gradient of a curve at a given point. On this sheet, we recap the definition of the derivative of a function in one variable and practise the method of differentiation from first principles.

We will also revise the standard formula for the derivative of a power, practice using it to differentiate polynomials. The problem set at the end of this resource includes some contextual questions to give a taste of how differentiation can be used to solve problems in the real world.

**First Principles Differentiation**

The rationale for this formula is the following: suppose is a second point on the curve, very close to .



Provided and are very close together, the line passing through both of these points will give a good approximation for the tangent line at .

Suppose and , where is a very small number.

The gradient of the straight line through and is given by the standard formula

As becomes very small, the point approaches , and becomes the tangent line at . The gradient of becomes the derivative of at , given by formula (1).

Differentiation using (1) is sometimes called *differentiation by first principles*.

Since we want to view the derivative as a function, we tend to use x instead of a in formula (1). Then, the derivative is given by

|  | (2) |
| --- | --- |

**Example 2.** Differentiate from first principles.

**Solution:** We use (2). Before we can evaluate the limit, we calculate for this particular function . First note that

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Letting tend to zero, we then get

**Exercise 3.**

1. Calculate .
2. Hence use (2) to differentiate from first principles.

**Some Properties of Derivatives**

**Linearity Property**

**Differentiating Powers**

Equation (3) may be used in conjunction with the linearity property for derivatives to differentiate any linear combination of powers of .

**Example 4.** Use equation (3) to differentiate .

**Solution:** First rewrite using power notation:

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By the linearity property for derivatives, we can calculate by differentiating term by term. Using (2), we get

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| --- | --- |
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**Exercise 5.** Use (2) to differentiate with respect to .

**Solutions to Exercises**

## Solution to Exercise 3:

1. We have and so

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| --- | --- |
|  |  |
|  |  |

1. Hence

## Solution to Exercise 5:

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